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PROPAGATION OF WEAKLY NONLINEAR WAVES OVER A RAMP

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The third order cubic Schrödinger equation (CSE) for weakly nonlinear water waves over slowly varying bottom is extended to include shoaling and next higher order variable depth effects. This involves a re-ordering of the dispersion relative to nonlinearity that further allowed a wider frequency bandwidth. The approach is to allow $\Delta\omega/\omega_o \sim \Delta k/k_o \sim O(\mu)$ with $\mu \sim O(\varepsilon^{2/3})$ where ε= wave steepness and with the depth variation $\Delta h/h \sim O(\mu^2)$. The evolution equation in terms of a slow time t_1(= $\mu\omega t$) and space x_1(= $\mu k_o x$) becomes

$$\frac{\partial A}{\partial t_1} + C_g \frac{\partial A}{\partial x_1} + \mu\left(-\frac{i}{2}\frac{\partial^2 \omega}{\partial k^2}\frac{\partial^2 A}{\partial x_1{}^2} - \frac{iC_g}{2k}\frac{\partial^2 A}{\partial y_1{}^2} + \frac{A}{2}\frac{\partial C_g}{\partial x_2} \right) +$$

$$+ \mu^2 \Big[-\frac{1}{6}\frac{\partial^3 \omega}{\partial k^3}\frac{\partial^3 A}{\partial x_1^3} + \left(\frac{C_g}{2k^2} - \frac{1}{2k}\frac{\partial^2 \omega}{\partial k^2} \right)\frac{\partial^3 A}{\partial x_1 \partial y_1{}^2}$$

$$+ i\left(\frac{\partial^2 \omega}{\partial k^2}\tanh q + \frac{C_g}{2k\tanh q} \right)\frac{\partial A}{\partial x_1}\frac{\partial q}{\partial x_2} + \left(-\frac{i}{2\omega} \right)\frac{\partial A}{\partial y_1}\frac{\partial (CC_g)}{\partial y_2}$$

$$+ \left(ik\frac{\partial \phi_0}{\partial x_1} - \frac{ik^2}{2\omega\cosh^2 q}\frac{\partial \phi_0}{\partial t_1} \right)\Bigg|_{z=0} A + \frac{i\omega k^2}{16\sinh^4 q}\left(\cosh 4q + 8 - 2\tanh^2 q \right)|A|^2 A \Big] = 0$$

In terms proportional to μ^2, the first two are asymmetric higher order linear dispersive terms, and the variable depth arises through terms involving the depth gradients and depth dependent coefficients. There are also terms from the wave induced current and the last term is the cubic nonlinear term.

The CSE predicts that the depth effect is restricted to shoaling that does not affect the stability features. However the higher order depth terms are shown to introduce higher order instability for all small wavelength disturbances beyond the Benjamin-Feir type as demonstrated by a linear instability analysis of shoaling Stokes waves. The propagation of uniform wavetrains over a variable depth consisting of a ramp is further analyzed by numerical solution of the extended CSE equation. Figure 1 shows the wave components of an initially uniform wavetrain at f_o= 1.5Hz (ν= 0), with a pair of linearly unstable sideband at Δf= ±0.11Hz (ν= ±1) and strength 0.1, and with initial steepness ε= 0.181. The variable depth consisted of a uniform deeper end at 45 cm (kh= 4.08) and a shallower end at 20 cm (kh= 1.89) with a ramp spanning τ (= $\mu^2 k_o x$) from 10.97 to 17.45. The evolution at the constant deeper depth of 45 cm is controlled by the alternate growth and decay of the unstable sideband and its unstable harmonic at $2\Delta f$ (ν= ±2) with quasi-recurrence. At the shallower end only the sidebands at Δf are unstable, though with significantly reduced growth and thus longer recurrence period. The evolution over the variable depth initially follows the constant depth case with the unstable sidebands growing at the deeper end. However the sidebands become stable with the decreasing depth though now with amplitudes comparable to the carrier. The spectrum at the shallower end becomes controlled by the wave components that are generated at the deeper end.

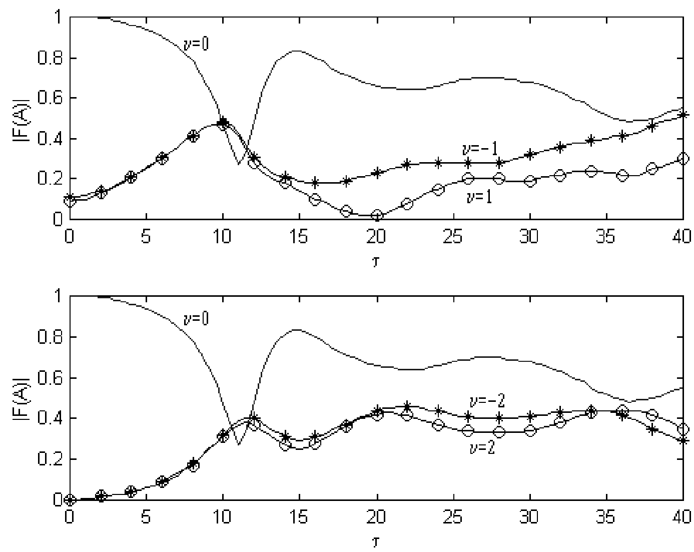

Figure 1 Spectral amplitudes of *A* for uniform wavetrain at f_0= 1.5Hz, ε = 0.181 and sideband disturbance strength of 0.1

Through this and other similar calculations, it is shown that the spectrum is controlled by the interplay of the distance to the ramp viz-a-viz the recurrence distance of the sidebands. The ramp is relatively short compared to the typical evolution/recurrence distance and thus essentially provides the "initial condition" for the spectrum propagating over the shallower end. Similar observations are also made for the case of another initial spectrum consisting of periodic groups.