

A Link Between “Universal” Fluctuations and Extreme Statistics ?

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Of necessity, experimentally accessible macroscopic variables usually average the effect of N fluctuating microscopic degrees of freedom. In equilibrium systems uncorrelated fluctuations are well known to sum to Gaussian form rapidly with N due to the central limit theorem. Current research interest in physics and for other complex systems, however, increasingly focuses on the non-Gaussian fluctuations seen in long-range correlated, turbulent or critical systems. Recent insight into these came from by Bramwell et al¹ who observed a distinctive skewed probability density with an exponential tail for fluctuations in both confined laboratory turbulence (in supplied power) and the XY model of critical phenomena (in magnetisation), subsequently reporting it in many other critical models such as sandpiles². We first briefly describe how the functional form they saw^{1,2} can be derived from a simple set of assumptions, motivating its “universality”. We then revisit the earlier conjecture by of Chapman et al that the similarity of Bramwell et al’s distribution to the Gumbel distribution of extreme statistics might be due to the largest event dominating the sum of fluctuations in patches of activity in a complex system. This is counter-intuitive, because of the behaviour of the short-ranged distributions with which we are more familiar, and is not ever exactly true. However we show it can be approximately true, particularly for spatially and/or temporarily long-range correlated fluctuations which effectively reduce the number of independent degrees of freedom of the system. We quantify this result by numerical simulations, extending the ideas of Romeo et al⁴ to the correlated case.

References

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